

### Math 2X03 - Homework 3

Due: May 24, 2016

Chapters Covered on Homework - 15.8-15.10, 16.1, 14.3, 14.5, 14.6

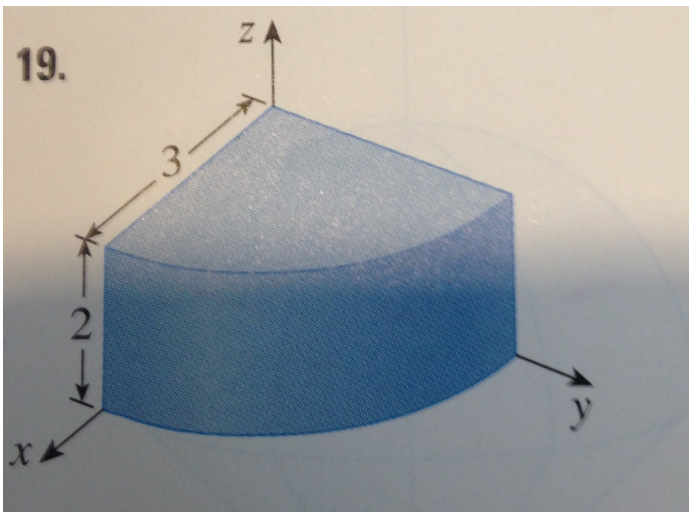
1. (Chapter 15.8 # 19) Evaluate  $\iiint_E (x + y + z) dV$ , where  $E$  is the solid in the first octant that lies under the paraboloid  $z = 4 - x^2 - y^2$ .
2. (Chapter 15.8 # 27) Find the mass and the center of mass of the solid bounded by  $z = 4x^2 + 4y^2$  and the plane  $z = a$  ( $a > 0$ ) if  $S$  has constant density  $K$ . (Hint: The region is homogenous and symmetric).
3. (Chapter 15.8 # 30) Evaluate the integral by changing to cylindrical coordinates:

$$\int_{-3}^3 \int_0^{\sqrt{9-x^2}} \int_0^{9-x^2-y^2} \sqrt{x^2 + y^2} dz dy dx$$

4. (Chapter 15.9# 13) Sketch the solid described by the given inequalities.

$$\rho \leq 1, \quad \frac{3\pi}{4} \leq \phi \leq \pi$$

5. (Chapter 15.9# 19) Set up the integral of an arbitrary continuous function  $f(x, y, z)$  in cylindrical or spherical coordinates over the solid shown below:



6. (Chapter 15.9# 23) Evaluate  $\iiint_E (x^2 + y^2) dV$ , where  $E$  lies between the spheres  $x^2 + y^2 + z^2 = 4$  and  $x^2 + y^2 + z^2 = 9$ .

7. (Chapter 15.9# 30) Find the volume of the solid that lies within the sphere  $x^2 + y^2 + z^2 = 4$ , above the  $xy$ -plane and below the cone  $z = \sqrt{x^2 + y^2}$ .
8. Use the change of variable formula to derive the formula for triple integration in cylindrical coordinates.
9. (Chapter 15.9# 39) Evaluate the integral by changing to spherical coordinates:

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy \, dz \, dy \, dx$$

10. (Chapter 15.10# 16) Use the given transformation to evaluate the integral

$$\iint_R (4x + 8y) \, dA,$$

where  $R$  is the parallelogram with vertices  $(-1, 3)$ ,  $(1, -3)$ ,  $(3, -1)$  and  $(1, 5)$ ;  
 $x = \frac{1}{4}(u + v)$ ,  $y = \frac{1}{4}(v - 3u)$ .

11. (Chapter 15.10# 21 a), b )

(a) Evaluate

$$\iiint_E dV,$$

where  $E$  is the solid enclosed by the ellipsoid  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ . Use the transformation  $x = ua$ ,  $y = vb$ ,  $z = cw$ .

(b) The earth is not a perfect sphere; rotation has resulted in flattening at the poles. So the shape can be approximated by an ellipsoid  $a = b = 6378$  km and  $c = 6356$  km. Use part (a) to estimate the volume of the earth.

12. (Chapter 15.10# 27) Evaluate the following integral by making an appropriate transformation

$$\iint_R e^{x+y} \, dA,$$

where  $R$  is given by the inequality  $|x| + |y| \leq 1$ .

13. (Chapter 14.3# 54) Find all the second partial derivatives of the function

$$f(x, y) = \sin^2(mx + ny)$$

14. Use the Chain rule to find  $\frac{\partial T}{\partial p}$ ,  $\frac{\partial T}{\partial q}$ ,  $\frac{\partial T}{\partial r}$  where

$$T = \frac{v}{2u + v}, \quad u = pq\sqrt{r}, \quad v = p\sqrt{qr}$$

15. (Chapter 14.6 # 14) Find the directional derivative of the function  $g(r, s) = \arctan(rs)$  at  $(1, 2)$  in the direction of the vector  $\mathbf{v} = 5\mathbf{i} + 10\mathbf{j}$ .
16. (Chapter 16.1 # 24) Find the gradient vector field of  $f(x, y, z) = x \ln(y - 2z)$ .
17. (Chapter 16.1 #25) Find the gradient vector field  $\nabla f$  of  $f(x, y) = x^2 - y$  and sketch it. .